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Aerodynamics III

Preliminary Tool for Quasi 1D Supersonic Wind Tunnel Analysis

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1 Introduction

Wind tunnels are machines used to study fluid dynamics in a laboratory environment, specifically interactions with man made objects such as structures, automobiles or aircraft. These facilities are able to create a controlled environment where engineers test the aerodynamic properties of objects under various conditions such as velocity, pressure and temperature.

The first wind tunnels were conceived at the end of the 19^{th} century as aeronautical research took its first steps. Their use was crucial to allow the design of heavier than air flying machines in the earlier years of the 20^{th} century. Supersonic wind tunnels were first conceived after the end of world war II as supersonic aircraft came into existence.

There are two main types of wind tunnels: continuous flow or intermittent. This report will only consider the continuous flow type which consist of a closed loop system where the fluid is accelerated using a compressor and excess heat generated is removed through a heat exchanger before the fluid passes through the test section where experiments take place.

In order to simulate supersonic flow, a convergent-divergent nozzle must be used to accelerate the fluid beyond mach 1, and since the compressors can only work under subsonic conditions, a diffuser must be present after the test section, to slow the fluid down to subsonic levels. The area of the throat of the diffuser can be variable in order to allow control over the position of a normal shock wave. In supersonic wind tunnels normal shock waves are a factor to consider and steps must be taken to ensure they remain in certain regions and do not affect important components.

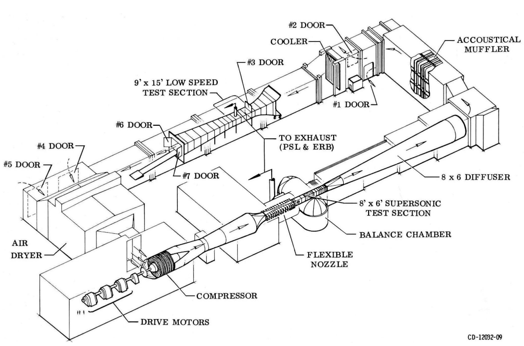

Figure 1: Supersonic wind tunnel schematic

The use of Computational fluid dynamics has revolutionised aerodynamics by allowing flow simulations around very complex geometries and has reduced the demand for wind tunnel testing, however, many study cases still benefit from their use.

An important task of supersonic wind tunnels is to test the aerodynamic properties of supersonic aircraft. Usually scaled down models are used in order to be able to use wind tunnels with modest size and power requirements. One such use case that is currently relevant is the development of the X-59 "Quesst" aircraft which is a super sonic test aircraft designed to implement ways of reducing the acoustic signature of sonic booms.

2 Objectives

The main focus of this project is to develop a user-friendly program designed to allow a user to simulate a quasi-1D approximation of a supersonic wind tunnel based on desired test section conditions and area as inputs.

Using this program a quasi-1D approximated model of a supersonic wind tunnel was simulated, based on the flight conditions of a known aircraft. Flow parameters along the tunnel sections and power requirements for the compressor were obtained for each one of the following operational instances:

- Ideal conditions:

In the ideal regime of a supersonic wind tunnel, the desired mach number is reached in the test section and an infinitesimal normal shock wave is present in the throat of the diffuser.

- Starting:

When starting the wind tunnel a shock wave will be present in the test section, which is undesired and must be addressed by increasing the pressure ratio of the compressor, causing the shock wave to move to the diffuser section, also referred to as "swallowing the shock".

- Operation:

In operating conditions, the whole test section is supersonic since the normal shock wave has moved to divergent section of the diffuser after "swallowing" the shock. The shock's position along this section with varying area will greatly influence the flow parameters.

3 Scope

To obtain the relevant parameters of the flow along the tunnel's length, we will resort to the isentropic relations and the normal shock equations to calculate pressure, temperature and density, mach number and area ratios. Only inviscid flow will be considered, so any stagnation pressure losses will come from the normal shocks, which equates to the necessary compressor power. Also, without friction, considering adiabatic flow and neglecting thermal contributions from the compressor there is no temperature gain that would have to be compensated by a heat exchanger.

The effect of the presence of a normal shock wave will be evaluated as a function of its position.

As a part of this project, an executable program was created which allows a user to choose a normal shock position and other operational conditions in order to obtain the relevant flow parameters.

To relate this project with real-world applications, the flight parameters of the X-59 aircraft will be used as inputs in the program and the corresponding flow parameters will be obtained.

4 Methotology and Relevant Equations

The following discussion on the design of a supersonic wind tunnel will have as its basis a convergent divergent nozzle, followed by a diffuser. This geometry is depicted in figure 2.

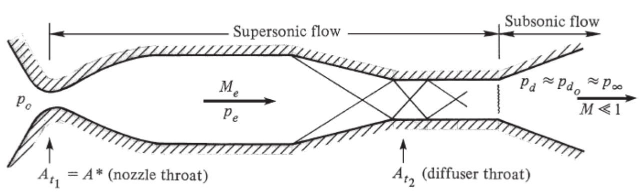

Figure 2: Nozzle with a conventional supersonic diffuser, from [1]

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The model used in this section is thus presented in Figure 3, where we have a convergent-divergent nozzle, followed by a test section and a diffuser in the form of a convergent-divergent nozzle. Only normal shocks are considered in this work and a quasi-1-D model of fluid flow is employed. When not cited, the relevant equations are retrieved from [2]. The flow is considered subsonic until Station 2. After Station 2 the flow is supersonic until a shock occurs. After a shock, the flow is considered subsonic.

As a first input of the analysis, the conditions of the test section are considered. The user can define M_3 (Mach at the test section), p_3 (pressure at the test section), T_3 (temperature at the test section), γ, the M_1 (Mach at the inlet section), nozzle length, test section length, and throat position at the nozzle.

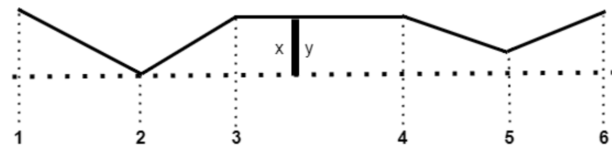


Figure 3: Wind Tunnel Diagram.

Using the conditions of the test section, it follows the calculation of the desired stagnation parameters, using Equations 1 to 3.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{1}$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \tag{2}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} \tag{3}$$

From the test section mach M_3, the following expressions are used to calculate the area ratio under operating conditions.

$$\frac{A_3}{A^*} = \frac{1}{M_3} \left(\frac{\frac{\gamma-1}{2}}{1 + \frac{\gamma-1}{2} M_3^2} \right)^{\frac{\gamma+1}{-2(\gamma-1)}} \tag{4}$$

From this area ratio and A_3, we can define the area of the first throat, since it will be constant throughout the analysis. We assume that the throat A_2 is always saturated, with a unitary Mach number. A_2 is defined by the following equation.

$$A^* = A_2 = \frac{A^*}{A_3} A_3 \tag{5}$$

To define A_1, and A_6, the first assumption is that those are equal. Then, from M_1, employing the same reasoning used in the A_2 calculation (Equations 4 and 5), A_1 and A_6 are calculated.

Next, area A5, in the diffuser throat, is calculated. Supposing the shock is localized between Stations 2 and 4, the geometry in those sections is already defined. In that case, the Mach number is calculated from the area at that position, using the following equation.

$$\frac{A_x}{A^*} = \frac{1}{M_x} \left(\frac{\frac{\gamma-1}{2}}{1 + \frac{\gamma-1}{2} M_x^2} \right)^{\frac{\gamma+1}{-2(\gamma-1)}} \tag{6}$$

After computing the Mach at that position, the ratio of stagnation pressure after and before the shock is computed. The higher the Mach number, the stronger will be the shock and this ratio approaches 1. The next equation provides the calculation method.

$$\frac{p_{0y}}{p_{0x}} = \left(\frac{\frac{\gamma+1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_x^2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{\gamma+1}{2\gamma M_x^2-(\gamma-1)}\right)^{\frac{1}{\gamma-1}} \tag{7}$$

Next, the ratio of critical areas present after and before the shock is computed.

$$\frac{A_y^*}{A_x^*} = \frac{p_{0x}}{p_{0y}} \tag{8}$$

By computing this last ratio, A_5 is computed, resorting to equation 9. A_5 will be equal or higher to the critical area after the shock, to avoid choking in the diffuser throat, depending on the user input. Although the area of the diffuser in practice cannot be the critical one, in the application created it can be set as critical and the mach will be unitary. However, due to the coding of the program, the flow after a shock never reaches supersonic conditions. The maximum area the user can input is the test section area A_3. This upper limit is the case for all shock locations.

$$A_5 >= A_y^* = \frac{A_y^*}{A_x^*}A_2 \tag{9}$$

In the case of a shock between Stations 4 and 5, the user is allowed to define A_5 with a minimum value of A_y^*, had the shock happened in the test section. The reason for this feature is that the calculation of a certain area, and consequently the critical area, requires a defined geometry, depending on the shock position. Thus, the minimum value chosen for A_5 is possible since the test section area will have the largest Mach, and thus the highest critical area, always preventing shocking on the diffuser throat. To define the matching A_y^* at a certain position with A_5, an iterative method, not employed in this work, would need to be used to match critical conditions to the minimum area allowed. After the geometry is defined, Equations 6 to 8 are used to calculate stagnation conditions change and the following equation to calculate the critical area after the shock.

$$A_y^* = \frac{A_y^*}{A_x^*}A_2 \tag{10}$$

If the shock is defined between Stations 5 and 6, the minimum area the user can choose is A_2. This is chosen to allow for the decrease of the diffuser throat until ideal conditions when the chock occurs at this position. Then Equations 6, 8, and 10 are used to calculate the critical area after the shock.

After calculating the shock conditions, the stagnation parameters after the shock are computed. The stagnation temperature across the shock is constant due to the adiabatic properties of the shock. The following equations state the stagnation properties change.

$$T_{0y} = T_{0x} \tag{11}$$

$$p_{0y} = \frac{p_{0y}}{p_{0x}}p_{0x} \tag{12}$$

$$\rho_{0y} = \frac{p_{0y}}{p_{0x}}\rho_{0x} \tag{13}$$

After the geometry is set and the critical areas calculated, the Mach number through all positions is calculated using Equation 6. Depending on the specific stagnation conditions, we can calculate all other fluid properties based on equations 1 to 3.

After the calculation of properties, the temperature at the inlet and outlet (Stations 1 and 6 respectively) of the wind tunnel, are accessible. Coupled with the mass flow rate, which can be calculated by Equation 14, the compressor power needed for the specific operation (Equation 15) can be estimated.

$$\dot{m} = \rho_2 U_2 A_2 \tag{14}$$

$$P = \dot{m}c_p(T_1 - T_6) = \dot{m}c_p T_6\left(\left(\frac{p_1}{p_6}\right)^{\frac{\gamma-1}{\gamma}} - 1\right) \tag{15}$$

From the computed pressure distribution, the ratio of pressures at the inlet and outlet can be computed.

$$R_p = \frac{P_1}{P_6} \tag{16}$$

A flowchart of the MATLAB code used in the previous methodology is in the Appendix in Figure 13.

A relevant parameter that may be retrieved after performing an analysis with a chock at the diffuser throat is the efficiency of the diffuser. This figure of merit is given by Equation 17. The upper term refers to the loss of stagnation pressure with a chock in the diffuser throat and the lower term is the equivalent stagnation pressure loss had the shock occurred at the test section [1]. In the context of this work, since only normal shocks are considered, the efficiency is equal to 1 if the diffuser throat is equal to the test section (virtually no diffuser) and increases for lower values of the throat area since the mach number diminishes, leading to a weaker shock. When the diffuser throat is equal to the first nozzle throat, the efficiency is maximum since the loss of stagnation pressure is minimal.

$$\eta_D = \frac{\left(\frac{p_{d0}}{p_0}\right)}{\left(\frac{p_{02}}{p_{01}}\right)_{normal\ shock\ at\ A_3}} \tag{17}$$

4.1 Quasi 1D Supersonic Wind Tunnel Application

As a final goal of the project scope, a standalone application was developed to provide an easy-to-use interface. Figure 4 is a general overview of the application, and some sections are relevant to mention.

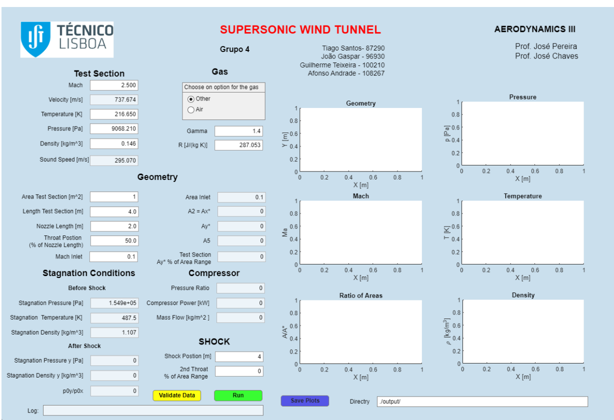


Figure 4: Interface developed for a easy-to-use interaction

The interface accepts some inputs from the user. The first region is called "Test Section" where the user can introduce some variables related with the test conditions, as mach, temperature, density, and pressure at which the test section should operate. The velocity and sound speed as just output variable to enlarge the problem understanding.

For the gas, the user can introduce manually the values for the gas constant, R, and the specific heat capacity ratio, γ. The user can also choose the option "Air" and default values for air are set.

The geometry of the tunnel is defined by two inputs as already explained. The inputs are the test section area and mach inlet. Other inputs, such as lengths, on the interface are just for geometry purposes and do not affect the performance since we are only considering isentropic conditions. A list of inputs and outputs of the interface are present in the Appendix, in Table B.

The "Stagnation Conditions" section just presents outputs, as well as the "Compressor" section, and details the previously presented parameters for a better understanding of the study. Also, critical and nozzle areas are also outputted.

For the shock two different inputs can be defined: the shock position, relative to the wind tunnel length; and the area of the second throat. These two parameters will help adjust what happens in terms of a normal shock wave. The first input sets the shock position, while the second input controls the area of the second throat, giving the user the possibility to choose an area in percentage from the critical value (0%) to the maximum area equal to the test section (100%), as presented in the methodology.

The user can find a plot region of the most important and relevant aspects of the wind tunnel analysis. Finally, to help write reports, prepare presentations, etc., it is possible to export the plots to a specified folder as ".png" images.

Warning: the program inputs are limited as empirical values and there is no guarantee that the output results make any sense. So it is important to validate the results.

5 Model Validation

In order to validate the scientific veracity of the program, it was used to resolve the exercise number 6 from [3] and then a comparison was performed between the computed results and the ones obtained at the class.

The exercise gives the conditions in the test section of the supersonic wind tunnel and asks to compute the diffuser and nozzle throat areas as well as to plot the mach number distribution with the operating conditions.

Mach Number	2.6
Pressure [kPa]	30
Temperature [K]	230
Area [cm^2]	180

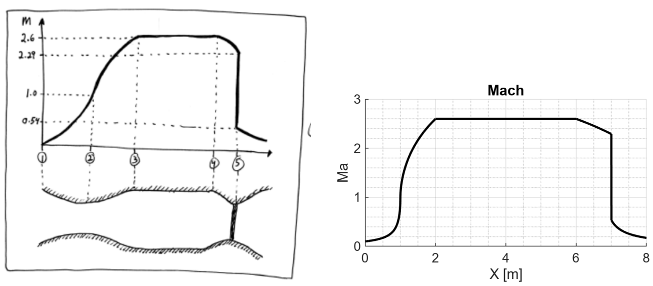
Table 1: Given test section conditions [3]

Then, using this data, the results can be computed making use to the above mentioned equations, isentropic and shock tables. The nozzle throat area is obtained using the equations 4 and 5. Then, the diffuser throat area value comes from the equations 8 and 9, for the case of starting condition. being A_5 defined as the equal to the critical area after the shock in the test section. Then, in operating conditions, to obtain the mach number at the diffuser throat the equation 10 needs to be used as well as the shock tables. After this step, the mach

number through the supersonic tunnel can be computed.
After presenting these results, the inputs are introduced in the developed application, and the results obtained in the application are compared and present in table 2. At Figure 5 there are depicted the mach distributions of the operating problem.

Parameter	Analitycal	Application
Nozzle throat area (A_2) [cm^2]	62.2	62.16
Diffuser throat area (A_5) [cm^2]	135.2	135.2
Diffuser throat mach before shock at operating conditions	2.29	2.29
Diffuser throat mach after shock at operating conditions	0.54	0.53

Table 2: Obtained results. [3]



(a) Mach number distribution plot obtained analytically. [3]

(b) Mach number distribution plot obtained by the application.

Figure 5: Mach number distribution at operating conditions.

Analyzing the obtained results, it is trivial to conclude that values computed from the different methods are very similar. This means the application is coherent with the class-obtained results and scientifically valid.

6 Results and discussion

The following data is obtained from the developed program, using as input parameters the flight conditions of the X-59 aircraft in table 3, which is designed to have a cruise mach number of 1.42 and a cruise altitude of 55000ft, from which was extracted ambient pressure, temperature and density using the international standard atmosphere data listed below. A test section of 1 m^2 was considered as a reasonable value for use with a scaled down model, a common practice for wind tunnel testing. The area of the second throat will always be set to the critical area for each scenario. (0% of area range, refer to B)

The following parameters in table 4 and the imposed position of the shock for each evaluated scenario will define the tunnel geometry.

Mach Number	1.42
Pressure [Pa]	9068.21
Temperature [K]	265.65
Sound speed [m/s]	295.07
γ	1.4
R [J/(Kgk)]	287.053

Table 3: Flow parameters for the test section

Test section area [m^2]	1
Test section length [m]	4
Nozzle length [m]	2
Inlet mach number	0.3

Table 4: Tunnel geometry parameters

6.1 Ideal case results

In this scenario a shock wave is imposed in the second throat (x=7m), which in this case is infinitesimal, having achieved the desired mach in the test section. As we can see from the results (Figure 6), the properties do not suffer any discontinuities along the tunnel, following isentropic relations since the normal shock is negligible and, also, since inlet and outlet areas are equal, properties of the flow at the outlet are the same as the inlet so no power is required from the compressor as no energy was lost due to the presence of shock waves. The areas of the throats are equal, and since the shock in the second throat is infinitesimal and can be considered isentropic, the critical area to achieve sonic mach at the throat of the diffuser is the same as the nozzle.

6.2 Starting regime results

When starting the tunnel, a shock wave is present in the test section. The shock's position was arbitrarily set at x=4m for our analysis. As seen in figure 7, at the shock's position there is a discontinuity in mach number, pressure, density and temperature characteristic of normal shock waves. The flow drops from the desired supersonic mach number to mach 0.73, accelerates up to mach 1 in the throat of the diffuser and drops to subsonic in the divergent. In this regime the area of the second throat is greater compared to the ideal regime as there is a non-negligible shock which causes stagnation conditions degradation requiring a larger critical area to have sonic conditions at the throat.

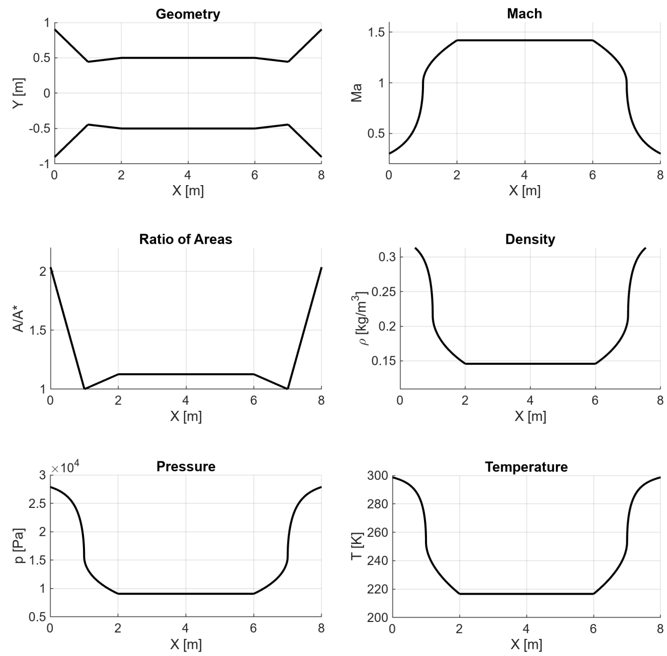


Figure 6: Tunnel geometry, mach number, Ratio of areas, density, pressure, and temperature distributions for ideal regime.

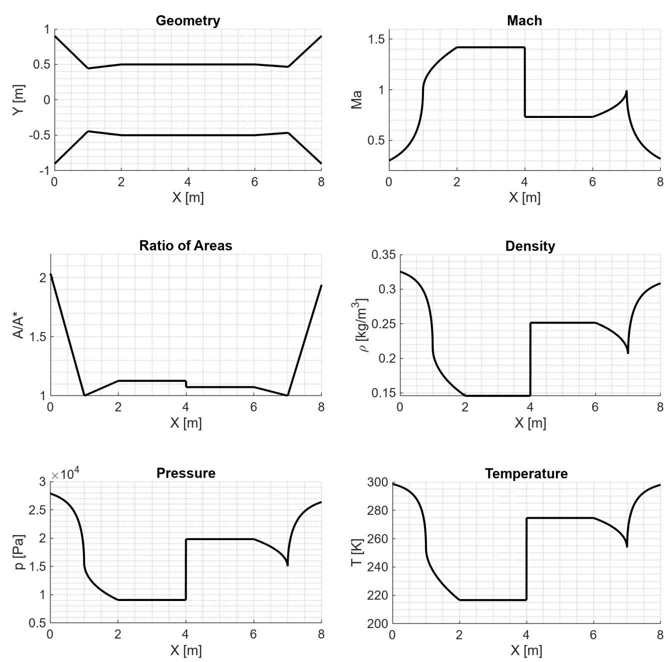


Figure 7: Tunnel geometry, mach number, Ratio of areas, density, pressure, and temperature distributions for starting regime.

6.3 Operational regime results

Imposing the shock's position at the diffuser's divergent section (x=7.5m) close to the throat, operating conditions are replicated. It can be seen in figure 8 that the fluid will slow in the convergent section of the diffuser, but, since the flow is still supersonic at the throat it will start to accelerate. The farther the shock is from the throat in the divergent section, the greater the mach number of the flow and stronger is the shock. Ideally, it should be located as close to the throat as possible reducing its strength, minimizing stagnation pressure losses, and thus, compressor power required. Another reason to minimize shock strength is to avoid unwanted stress on the structure and other components, which can be considerable. Although it falls outside of the scope of this project since inviscid flow was considered, having a stronger shock could be an advantage as the fluid velocity will be lower downstream of the shock resulting in lower losses due to friction. Since the normal shock is not situated between the two throats, their critical areas are equal, and A_5 is defined with this value, since the flow between them is isentropic. As the flow is still supersonic in the second throat, its area could be larger and still be able to maintain the shock in the divergent section. In real situations, this diffuser throat needs to be slightly larger than critical to prevent choking.

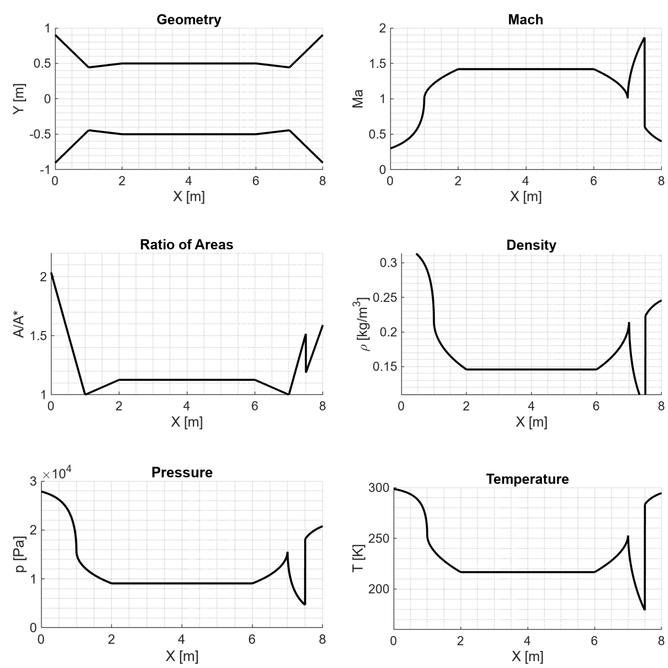


Figure 8: Tunnel geometry, mach number, Ratio of areas, density, pressure, and temperature distributions for operational regime.

6.4 Compressor results comparison

From the compressor data in Table 5 it can be seen that the shock's position will greatly influence the compressor requirements. Across all 3 regimes the mass flow remains constant, since they all present chocked flow, with sonic condi-

Regime	Compressor Power [kW]	Pressure ratio
Ideal	0	1
Starting	36.99	1.057
Operational	248.3	1.342

Table 5: Compressor power and Pressure ratio

tions at the nozzle throat. In the ideal case there are no losses and so no compressor power is required, whereas in the other scenarios, the stagnation conditions degradation due to the shock will have to be compensated by the compressor for the given regime to be maintained. In order to move the shock from the test section to the diffuser, or to "swallow" the shock, the compressor must input more power. In normal operation, depending on the shock position, the power required to maintain the shock can be higher than the test section if the outlet area is too large, as it is in this situation. A deeper insight into this phenomenon is given in the next section. For demonstration purposes the shock was placed at x=7.5m, which is why compressor power is so high for this case. In reality, the shock would remain much closer to the throat requiring much less compressor power.

6.5 Compressor Analysis

Understanding the operational mode of a supersonic tunnel involves significant consideration of the compressor's capacity to deliver sufficient power to overcome the pressure ratio between the inlet and outlet (points 1 and 6), as defined by Equation 16. The relationship between compressor power and the temperature difference between the inlet and outlet, as indicated by equation 15, is crucial. The temperatures at both points are determined from the isentropic relations, as the stagnation temperature remains constant throughout the normal shock wave phenomena. With this, one can calculate all the numerical values for the compressor and evaluate the impact of the shock wave on its performance.

Three different analyses were conducted regarding the influence on compressor power, depending on shock position. Firstly, the second throat area, A_5, was kept constant throughout the evaluation of the shock in all positions from the first throat (Station 2) to the end of the diffuser outlet section (Station 6). The chosen area is the critical area after a shock has occurred at the test section. The second analysis was performed considering the same range for the shock position but with the minimum possible value for A_5 at each section. Minimum values for this area are described in the methodology section. Both this analysis assumed the inlet and exit areas equal to that of the test section. A third analysis was then performed, using also the minimum diffuser areas, but with inlet and exit areas defined by the inlet mach. The conditions are the same present in table 3, and 4.

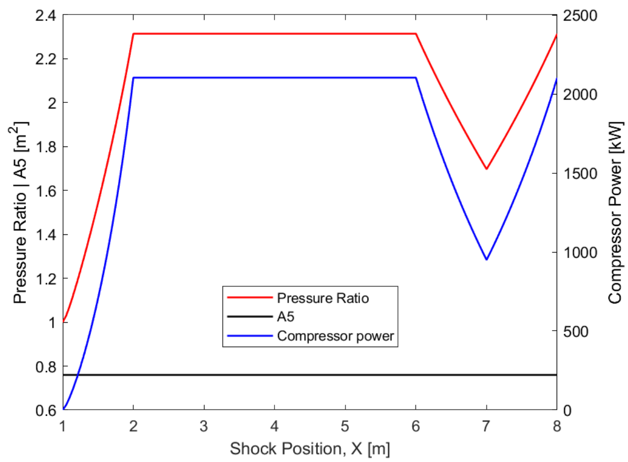


Figure 9: Compressor pressure ratio and power for different shock positions considering the same A_5 area

Figures 9 and 10, corresponding to the first and second analysis, seem to have the same behavior, however, some details are different and essential to understanding the impact of the normal shock wave. Both figures show that, at $x = 1m$ (first throat position), the compressor power is zero. This is a valid result as the shock tends is occurring in a convergent-divergent nozzle throat, leading to an infinitesimal normal shock and then a pressure ratio of 1, meaning no compressor power is needed. For the second throat it does not happen in both cases. Considering a constant area, A_5, figure 9 shows that, for the second throat position ($x = 7m$) neither the pressure ratio is 1 or the compressor power is 0. This is due to the A_5 area value is above the critical area (equal to A2) for the second throat and then sonic conditions are not achieved. The normal shock at the second throat is strong (large pressure ratio), meaning stronger than an infinitesimal normal shock wave. On the other hand, it occurs in figure 10 once the A_5 at $x = 7m$ is the critical area, so sonic conditions are achieved, and an infinitesimal chock occurs $\frac{p_{0y}}{p_{0x}} = 1$.

As expected, if the normal shock wave happens inside the test section (from point 3 to point 4, or from $x = 2m$ to $x = 6$), the position is not relevant. This is a valid result once in this quasi-1D model the compressor power is only affected by the temperature difference between the inlet and outlet. The equation 7 expresses the stagnation pressure ratio across the shock as a function of mach, and as can be concluded the $\frac{p_{0y}}{p_{0x}}$

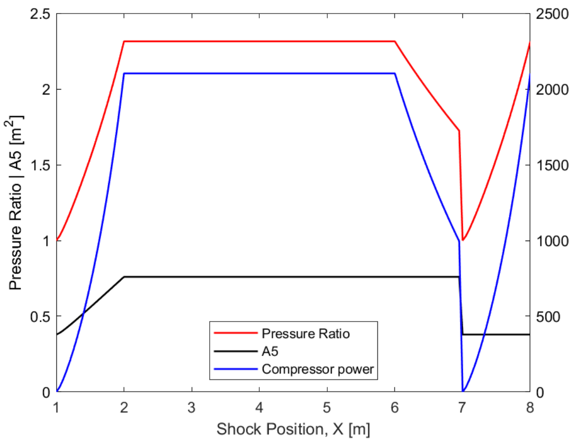


Figure 10: Compressor pressure ratio and power for different shock positions considering the minimum possible A_5 area

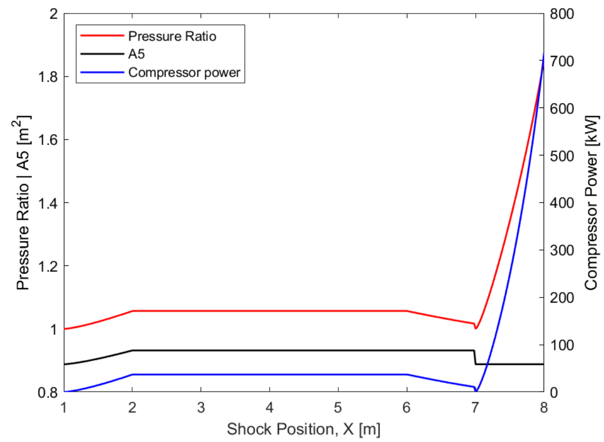


Figure 11: Compressor pressure ratio and power for different shock positions considering the minimum possible A_5 area, and inlet and exit areas defined by the inlet mach.

is the same since the same area is kept along the test section length.

As already mentioned, the equation 7 expresses the pressure ratio across the shock as function of mach, and it is important to highlight that in the divergent and convergent part of the first nozzle and in the diffuser, the compressor power varies with the shock position because the mach number changes.

Finally, it is important to understand what are the differences between the shock in the convergent or divergent part of the diffuser in figure 10 because the behavior is not symmetric over the throat. The discontinuous drop in power is caused due to setting the minimum area at the diffuser throat. This feature is explained in the methodology section. So, from these results, it can be observed that by having a variable diffuser throat, the required power in operation can be set to a minimum by decreasing the diffuser throat after the shock is swollen.

For the third analysis, results are presented in Figure 11. Remember that the conditions that change in this situation, regarding the second analysis (Figure 10) are the areas of the inlet and outlet. From the definition of the inlet Mach number, the inlet and outlet area results in these areas being larger than the test section areas. if a shock is to occur near the exit, this means that the supersonic mach number in this region is higher than the test section, leading to stronger shocks and consequently to higher compressor powers. So, when considering the design of a supersonic tunnel the outlet area will influence farther shocks and the compressor power required to hold shocks near the exit will steply increase with larger outlet areas. This further stresses the need for the shocks to be located near the throat of the diffuser, minimizing the shock and reducing power.

6.6 Diffuser Efficiency

To further extend the statement of the importance of the variable throat area at the diffuser, a diffuser efficiency study is performed, resorting to the conditions of Table 3 and 4. The diffuser efficiency is present in Equation 17 and takes into account the loss of stagnation pressure in the diffuser, in relation to stagnation pressure loss when a shock occurs at the test section. This analysis is performed with a shock at the diffuser throat, while varying difuser throat area and depicted in Figure 12. There are two vertical lines, representing the condition at which the diffuser is chocked ($A_5 = A_2$) and the starting area, corresponding to the critical area after the shock, when it occurs in the test section.

It can be observed that the efficiency of the diffuser is higher when the throat decreases. When the throat decreases, the mach number at the throat also decreases, leading to weaker shocks and higher efficiency values, concluding once again the importance of the variable throat area in reducing shock intensity and compressor power.

7 Conclusions

In conclusion, this work objectives were achieved, providing a computational tool for aiding the preliminary design of a supersonic wind tunnel. The relevant equations and implementation are presented, including an interactive interface for easy use by a user. A validation exercise was followed, in-

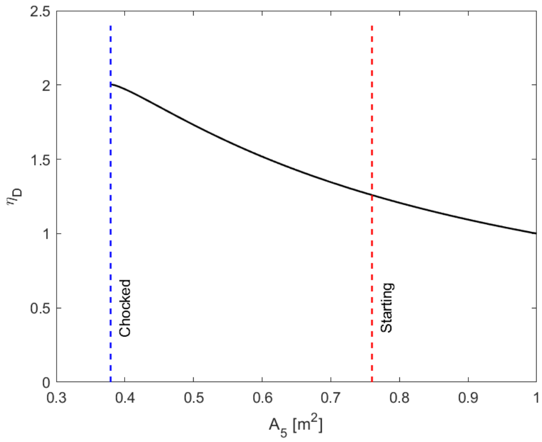


Figure 12: Diffuser efficiency study.

dicating the program's physical coherence. The design and results of a supersonic wind tunnel were also accomplished, based on the x-59 aircraft use case. The tunnel was analysed for the ideal, starting, and operational conditions, leading to a better understanding of the physical processes involved in the design of a supersonic wind tunnel. This design was also inquired regarding compressor power, depending on the shock position, and diffuser throat efficiency, stating the importance of the variable throat area for efficient operation.

Bibliography

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[2] J. C. F. Pereira, *Aerodinâmica Compressível*. Instituto Superior Técnico: AEIST, 2021, vol. 1.

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Appendix

A Program Flowchart

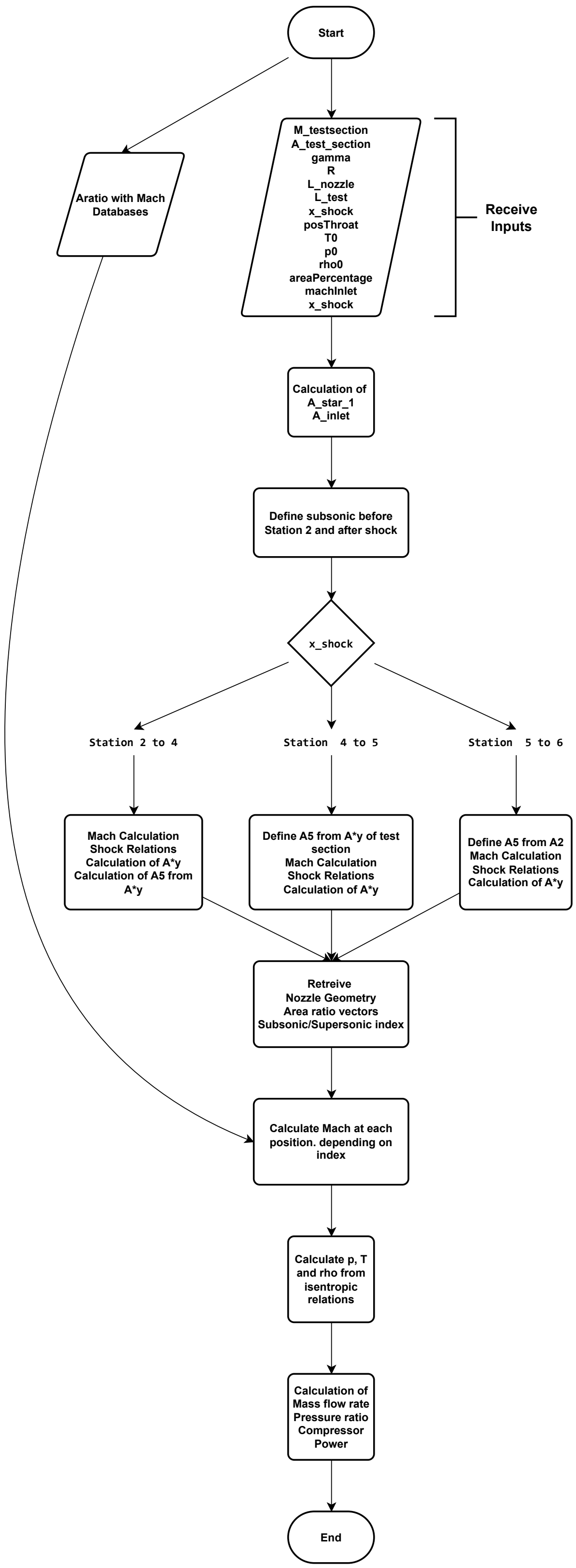

Figure 13: Flowchart of the program.

B User Manual

Input	Description	Range	Output	Description
Mach	Mach of test section	1-5	Velocity [m/s]	Calculated fluid speed at test section
Temperature [K]	Temperature of test section	>0	Sound Speed [m/s]	Calculated sound speed at test section
Pressure [Pa]	Pressure of test section	>0	Area inlet	Inlet Area. Equal to exit area
Density [kg/m^3]	Density of test section	>0	A2 = Ax*	Area of first nozzle. Equal to Ax*
Gamma	Gamma of fluid	>0	Ay*	Critical area after shock
R [J/(kg K)]	Gas constant of fluid	>0	A5	Area of the diffuser
Area Test Section [m^2]	Area of the test section	>0	Test Section Ay* % of Area Range	The area range percentage, corresponding to the Ay* of test section.
Length Test Section [m]	Length of test section	>4*Area Test Section	Stagnation Pressure [Pa]	Stagnation pressure before shock
Nozzle Length [m]	Length of Nozzle	>2* Test Section	Stagnation Temperature [K]	Stagnation temperature before shock
Throat Position (% of Nozzle Length)	Throat position in relation to nozzle	20-80	Stagnation Density [kg/m^3]	Stagnation density before shock
Mach Inlet	Mach of the inlet	0-1 (The program limits this value outputing and error, if after shock the Area ratio goes below 1 at any position)	Stagnation Pressure y [Pa]	Stagnation pressure after shock
Shock Position	Position of the shock	Station 1 to Station 6	Stagnation Density y [kg/m^3]	Stagnation density after shock
2nd Throat % of Area Range	Percentage from allowable area range, outputing A5	0-100	p0y/p0x	Ratio of stagnation pressures
			Pressure ratio	Ratio of exit and inlet pressure
			Compressor Power [kW]	Compressor required power
			Mass Flow [kg/s]	Mass flow rate